

A Transfer Matrix Method for Resonances in Randall-Sundrum Models III: An analytical comparison.

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Abstract

The transfer matrix method is used to analyze resonances in Randall-Sundrum models. Although it has successfully been used previously by us we provide here a comparison between the numerical and analytical models. To reach this we first find new exact solution for the scalar, gauge, Kalb-Ramond and q -form fields. Then we calculate numerically the resonances by the transfer matrix method and compare with the analytical result. For completeness, this is done for models with and without the dilaton coupling. The results show a perfect agreement between the analytical and numerical methods.

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1 Introduction

One of the main models treating aspects of physics of extra dimensions is the Randall-Sundrum model [1, 2]. This model gives a possible solution to the hierarchy problem and tell us how gravity is trapped in our four dimensional world. However, the lack of a more physical application have posed some challenges to this model through the past years. One of its problems is the appearance of spacetime singularities due to the presence of infinitely thin membranes. In such case, more realistic models based on smooth solutions representing the four-dimensional membrane embedded in a higher dimensional spacetime is needed. In special cases, where the membranes are generated by scalar fields, its is very simple to obtain solutions through the superpotential method. In particular, several membrane types were considered: those generated by models with more than one scalar fields, deformed membranes and so on [3–9]. Beyond that, the analysis of how the localization is made for various fields (tensors and spinors) have been understood [10–17]. Other aspects include discussions about the tensions of membranes [18, 19].

Another interesting problem is related to understand details of the interactions between the membrane and several particles that are not the usual zero modes. In other words, the approach to this problem considers the computational aspects of resonances for various models. In all of the models the massive spectrum is determined by a Schrödinger like equation with a potential that falls to zero at infinity. The spectrum is not discrete and we have a ill defined effective action. Despite of this, just like in the case of semiconductors heterostructures, there is the possibility of appearance of resonances. This analysis have been done extensively in the literature [5, 14, 15, 22–33]. In order to analyze resonances, we must compute the transmission coefficient (T), that gives a clear physical interpretation of what happens to a free wave interacting with the membrane. The idea of the existence of a resonant mode is that, for a given mass, the transmitted and reflected oscillatory modes are in phase inside the membrane. The transmission coefficient has a peak at that mass value, meaning that the amplitude of the wave-function has a maximum value at $z = 0$ and the probability to find this KK mode inside the membrane is higher. This method has been used previously to analyze resonant modes of gravity, fermion and form fields [34, 35].

The background considered here consists of a symmetric Z_2 thick domain wall interpolated between two BPS vacua. The scalar field is important because, in some cases, its behaviour is very similar to the gravitational field's behaviour [37]. The gauge vector field is an important ingredient of the standard model and, even being a not localizable field in theories with conformal symmetries, we may understand how its resonances appear due to the specific brane chosen. In this case, we consider the corrections due to a dilaton field coupled to the gauge field and others antisymmetrical tensors fields. These antisymmetric tensor fields arise quite naturally in string theory [38, 39] and supergravity [40] and play an important role in the dualization processes [41, 42]. In particular they appear in the $R - R$ sector of each of the type II string theories. These tensor

fields couple naturally to higher- dimensional extended objects, the D -branes, and are important for their stability. From a more mathematical point of view, they are related to the linking number of higher dimensional knots [43]. The rank of these antisymmetric tensors is defined by the dimension of the manifold [44]. Beyond this, these kind of fields play an important role in the solution of the moduli stabilization problem of string theory [45–47].

For the physics of extra dimensions is important to study higher rank tensor fields in membrane backgrounds. In this context, the antisymmetric tensor fields have already been considered in models of extra dimension. Generally speaking, the q -forms of highest rank do not have physical relevance because when the rank increases the number of gauge freedom increases as well. Such fact can be used to cancel the dynamics of the field in the brane [48]. The spectrum mass of the two and three-form have been studied in Refs. [49] and [50] in the context of five dimensions with codimension one. The coupling between two and three-forms with the dilaton was also studied but in a different context [51–53]. An analysis of localization and computations of resonances due to q -forms have been previously made by the authors in [54].

Here, in this piece of work we show results of particular importance to situations where the gravity backgrounds are smooth generalizations of the Randall-Sundrum model and procedures that are applied even when the Schrödinger potential is not known analytically [20, 21]. More precisely, we continue the analysis made in [36] in order to understand the behaviour of scalar, gauge vector fields and more general q -forms in a model that can be solved analytically. Another goal is to make a comparison between the calculation obtained through the solution of the analytical method and the solutions obtained by the method of transmission coefficients. The importance of such a model with analytical solution resides in the fact that in the models with thick membranes their thickness is a constant parameter. When the thickness is parametrized, there is a way to obtain the thin limit and the thick membrane limit. In this sense, the physics related now can be understood by means of a more general situation that can mimics several types of membranes. Through this comparison process we can establish the Transfer Matrix method, once an for all, as a tool to understand some physical aspects of extra dimensions.

The paper is organized as follows. In the first section we review the method of computation of transmission coefficients in order to make comparisons between different formalisms. In the second section we present the general procedure to solve the Schrödinger equation coming from models of extra dimensions and solve it for the scenario proposed. In the third section we quickly remember the gravitational results and make a generalization regarding solutions as plane waves far from the membrane. The remaining sections are devoted to studies about the several bosonic fields, i.e., the antisymmetric gauge fields that usually appear in literature. They are the scalar field, gauge vector and tensor (Kalb-Ramond) fields, besides more general q -forms. Finally we present our conclusions and perspectives of work.

2 A Simple Example

In this section we give a review of the Transfer Matrix method and present a model with known analytical solution. After that we compare results of both the analytical and numerical transmission coefficient computations. The analytical model discussed here is already known and can be found in [55]. It is important to say that all potentials considered in this manuscript are volcano like, see Fig. 1.

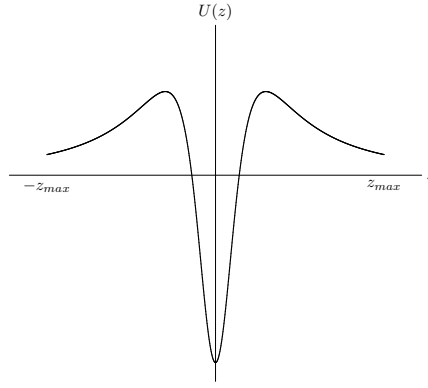


Figure 1: General potential with parity symmetry with $\lim_{z \rightarrow \pm\infty} U(z) = 0$.

The potential given in Fig.1 can be approximated by a series of potential barriers. In each region showed in the Fig.2 the Schrödinger equation can be

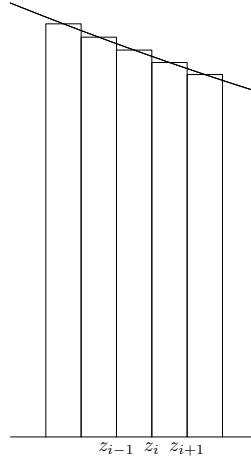


Figure 2: The multistep regions.

solved for the interval $z_{i-1} < z < z_i$, where the potential can be approximated

by

$$U(z) = U(\bar{z}_{i-1}) = U_{i-1}, \quad \bar{z}_{i-1} = (z_i + z_{i-1})/2. \quad (1)$$

As the potential is null in the infinity the solution must be a plane wave. Then, as in Fig.3, we consider a plane wave colliding with the membrane. This

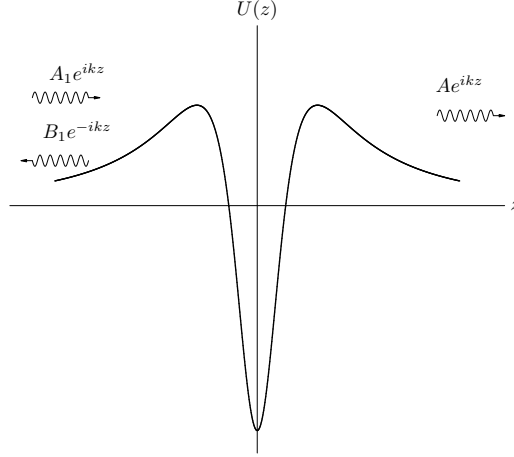


Figure 3: The scattering of incident wave into reflected and transmitted waves.

solution choice is very important for the analytical solution considered in the next section. As the final goal here is to compare the analytical and the numerical results, we will consider plane waves as boundary conditions to our analytical solution. Therefore, the solution in each interval is

$$\psi_{i-1}(z) = A_{i-1}e^{ik_{i-1}z} + B_{i-1}e^{-ik_{i-1}z}, \quad k_{i-1} = \sqrt{\lambda - U_{i-1}}, \quad (2)$$

and the continuity of the $\psi_{i-1}(z)$ and $\psi'_{i-1}(z)$ at $z = z_i$ gives us

$$\begin{pmatrix} A_i \\ B_i \end{pmatrix} = M_i \begin{pmatrix} A_{i-1} \\ B_{i-1} \end{pmatrix}. \quad (3)$$

In the above equation we have that

$$M_i = \frac{1}{2k_i} \begin{pmatrix} (k_i + k_{i-1})e^{-i(k_i - k_{i-1})z_i} & (k_i - k_{i-1})e^{-i(k_i + k_{i-1})z_i} \\ (k_i - k_{i-1})e^{i(k_i + k_{i-1})z_i} & (k_i + k_{i-1})e^{i(k_i - k_{i-1})z_i} \end{pmatrix} \quad (4)$$

and performing this procedure iteratively we reach to

$$\begin{pmatrix} A_N \\ B_N \end{pmatrix} = M \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}, \quad (5)$$

where,

$$M = M_N M_{N-1} \cdots M_2 M_1. \quad (6)$$

The transmission coefficient is therefore given by

$$T = 1/|M_{22}|^2. \quad (7)$$

In order to numerically obtain resonances we choose z_{max} to satisfy $U(z_{max}) \sim 10^{-4}$ and let m^2 runs from $U_{min} = U(z_{max})$ to U_{max} (the maximum potential value). We divide $2z_{max}$ by 10^4 or 10^5 such that we have $10^4 + 1$ or $10^5 + 1$ transfer matrices.

Now we turn our attention to a model with analytical solution. Consider a Schrödinger equation with a potential given by [55]

$$U(z) = \frac{U_0}{\cosh^2 \alpha z}, \quad (8)$$

with profile as in Fig. 4.

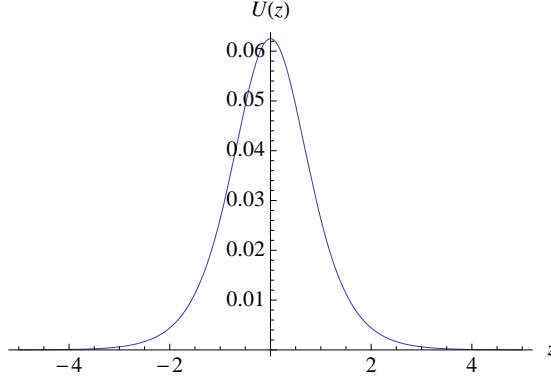


Figure 4: The profile of the potential in Ref. [55] with $U_0 = 1/16$ and $\alpha = 1$.

The analytical solution to the wave function for the potential given by Eq. (8) is

$$\psi = (1 - \xi^2)^{-ik/2\alpha} F\left(-i\frac{k}{\alpha} - s, -i\frac{k}{\alpha} + s + 1, -i\frac{k}{\alpha} + 1, \frac{1}{2}(1 - \xi)\right), \quad (9)$$

where

$$\begin{aligned} \xi &= \tanh \alpha z, & k &= \sqrt{2mE}/\hbar, \\ s &= \frac{1}{2} \left(-1 + \sqrt{1 - \frac{8mU_0}{\alpha^2 \hbar^2}} \right). \end{aligned} \quad (10)$$

As stressed in [55] this solution has the desired asymptotic plane wave form. After some calculations we can find the transmission coefficient:

$$\begin{aligned}
T &= \frac{\sinh^2(\pi k/\alpha)}{\sinh^2(\pi k/\alpha) + \cos^2[\frac{1}{2}\sqrt{1 - 8mU_0/\hbar^2\alpha^2}]}, & 8mU_0/\hbar^2\alpha^2 < 1, \\
T &= \frac{\sinh^2(\pi k/\alpha)}{\sinh^2(\pi k/\alpha) + \cosh^2[\frac{1}{2}\sqrt{8mU_0/\hbar^2\alpha^2 - 1}]}, & 8mU_0/\hbar^2\alpha^2 > 1. \quad (11)
\end{aligned}$$

Now we can compare the results obtained by the analytical and numerical calculations. We use $\alpha = \hbar = m = 1$ and $U_0 = 1/16$. Fig. 5 shows: (a) The graphic for the analytical transmission coefficient T_A , (b) The graphic for the numerical transmission coefficient T_N and (c) The graphic with the ratio T_N/T_A . There is no distinction between both methods, the agreement is almost complete.

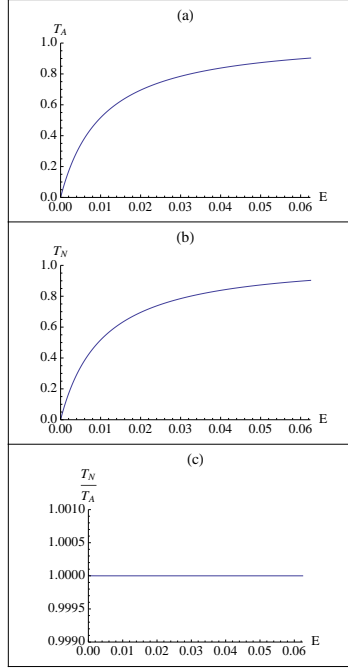


Figure 5: Transmission coefficients plots in the Landau book example: (a) The analytical transmission coefficient T_A , (b) The numerical transmission coefficient T_N and (c) The ratio T_N/T_A .

3 The General Analytical Solution

In this section we show the procedure used through all this work. In models describing interactions between fields and membranes, the equations of motion

coming from the extra-dimension dependence plays an important role, and have a Schrödinger like final form. To arrive at that this result we provide a short review here (details can be found in [37]). First of all we consider the line element of the background space as

$$ds^2 = e^{2A(y)}\eta_{\mu\nu} + e^{2B(y)}dy^2, \quad (12)$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. This metric comes from the inclusion of the interaction produced by a dilaton field that, in a specific model, gives rise to localization of gauge fields.

The Einstein's equations give the relation $B = A/4$ between the functions $A(y)$ and $B(y)$ [37]. These equations are obtained from the usual superpotential method in models containing kink like defects. It is important to cite that the dilaton field have been already been treated in the Randall-Sundrum scenario [53]. As we are going to consider models with and without the dilaton ($B = 0$), we introduced a constant b to take account of this and we get $B(y) = (1-b)A(y)$. The value of b with the dilaton is $3/4$ and 1 without the dilaton. Using this metric background, in general, the equations we need to work have the following form

$$\left(-\frac{d^2}{dy^2} + P'(y)\frac{d}{dy} + V(y)\right)\psi(y) = m^2 Q(y)\psi(y), \quad (13)$$

where $P(y) = \gamma A(y)$, $Q(y) = e^{-2bA(y)}$ and $V(y) = 0$ for all fields except gravity, in which $V(y) = 2A''(y) - 2(1+b)A'(y)^2$. Here, γ is a constant that depends on the field. We can transform (13) in a Schrödinger like equation through the transformations

$$\frac{dz}{dy} = f(y), \quad \psi(y) = \Omega(y)\bar{\psi}(z), \quad (14)$$

with

$$f(y) = \sqrt{Q(y)}, \quad \Omega(y) = \exp(P(y)/2)Q(y)^{-1/4}, \quad (15)$$

and

$$\bar{U}(z) = V(y)/f^2 + (P'(y)\Omega'(y) - \Omega''(y))/\Omega f^2, \quad (16)$$

where the prime means the derivative in y . We emphasize here, that the above expression is useful when $dz/dy = f(y)$ do not have an analytical solution. When this solution is known it is better to express the potential in terms of derivatives in the z coordinate:

$$\bar{U}(z) = \bar{V}(z)/\bar{f}^2(z) + \frac{\bar{P}'(z)\bar{\Omega}'(z) - \bar{\Omega}''(z)}{\bar{\Omega}(z)} - \frac{\bar{\Omega}'(z)}{\bar{\Omega}(z)}\frac{\bar{f}'(z)}{\bar{f}(z)}, \quad (17)$$

where $f(y) = \bar{f}(z)$. When the above steps are performed we get a Schrödinger like equation

$$\left\{-\frac{d^2}{dz^2} + \bar{U}\right\}\bar{\psi}(z) = m^2\bar{\psi}(z), \quad (18)$$

with potential $\bar{U}(z)$ given by

$$\bar{U}(z) = c\bar{A}''(z) + c^2(\bar{A}'(z))^2, \quad (19)$$

where $c = -(\gamma + b)/2$ for all fields and $c = 3/2$ for gravity.

An analytical solution has been obtained previously for the gravity case in [36] but without the dilaton coupling. We use the same method to solve the case with the dilaton coupling and for other fields. The relation between the z and y coordinates in all cases is given by $\frac{dz}{dy} = e^{|y|} = e^{-b\bar{A}(z)}$, for $|z| > d/2$, where d is a constant. With this, we obtain

$$A(y) = \bar{A}(z) = -\frac{1}{b} \ln(|z| + \beta). \quad (20)$$

The above expression determines the Schrödinger equation potential in the region $|z| > \frac{d}{2}$ as

$$\bar{U}(z) = \frac{a}{(|z| + \beta)^2}, \quad (21)$$

with $a = \frac{c}{b} + \frac{c^2}{b^2}$. Since we can have resonance only for positive potential, we restrict a to be positive, i. e, $c/b > 0$ or $c/b < -1$. For the region $|z| \leq d/2$, we choose

$$A(z) = \frac{1}{c} \ln \cos(\sqrt{V_0}|z|), \quad (22)$$

such that

$$\bar{U}(z) = -V_0, \quad |z| \leq d/2. \quad (23)$$

Continuity of the metrics and its derivative at $z = \pm d/2$ give us the relations

$$\begin{aligned} \left(\frac{d}{2} + \beta\right) &= \cos(\sqrt{V_0}\frac{d}{2})^{-b/c}, \\ \left(\frac{d}{2} + \beta\right)^{-1} &= \frac{b}{c}\sqrt{V_0} \tan(\sqrt{V_0}\frac{d}{2}). \end{aligned}$$

In order to obtain V_0 , β and d we will introduce the parameter $x = d\sqrt{V_0}/2$ as in [36].

For the region $|z| > d/2$, the Schrödinger equation can be solved if we consider $\bar{\psi}(z) = \sqrt{|z| + \beta}\bar{\phi}(z)$. With this transformation we obtain

$$\sqrt{|z| + \beta}\bar{\phi}'' + \frac{1}{\sqrt{|z| + \beta}}\bar{\phi}' + \sqrt{|z| + \beta} \left[m^2 - \frac{1}{4(|z| + \beta)^2} - \frac{a}{(|z| + \beta)^2} \right] \bar{\phi} = 0. \quad (24)$$

Multiplying now by $(|z| + \beta)^{3/2}$ and defining $u = m(|z| + \beta)$ we arrive at

$$u^2\bar{\phi}'' + u\bar{\phi}' + [u^2 - \nu^2] \bar{\phi} = 0, \quad (25)$$

where the prime means a u derivative and $\nu^2 = (\frac{1}{2} + \frac{c}{b})^2$. This is a Bessel equation of order ν . Here we are interested in solutions behaving like plane waves when $z \rightarrow \pm\infty$. Solutions with this properties are given by

$$\begin{cases} H_\nu^{(1)}(u) &= J_\nu(u) + iY_\nu(u), \\ H_\nu^{(2)}(u) &= J_\nu(u) - iY_\nu(u), \end{cases} \quad (26)$$

with asymptotic behavior

$$\begin{cases} \lim_{z \rightarrow \infty} H_\nu^{(1)}(u) &= \sqrt{\frac{2}{\pi u}} e^{i(z - \frac{\pi}{4} - \frac{\nu\pi}{2})} \\ \lim_{z \rightarrow \infty} H_\nu^{(2)}(u) &= \sqrt{\frac{2}{\pi u}} e^{-i(z - \frac{\pi}{4} - \frac{\nu\pi}{2})}. \end{cases} \quad (27)$$

The choice of asymptotic plane waves is related to the fact that we want to compare this solution with the transfer matrix method, where a plane wave colliding with the brane is used. It is important to stress here that this is different from the solution found in [36], where an exact solution was used but no plane wave was considered. Using this consideration, the solution is given by

$$\psi(z) = \sqrt{\frac{u}{m}} \left(AH_\nu^{(1)}(u) + BH_\nu^{(2)}(u) \right), \quad (28)$$

with the desired behavior.

$$\lim_{z \rightarrow \infty} \psi(z) = \sqrt{\frac{2}{m\pi}} (Ae^{imz} + Be^{-imz}). \quad (29)$$

In the region $|z| \leq d/2$, we have the solution

$$\psi_I(z) = ae^{iKz} + be^{-iKz}, \quad (30)$$

with $K = \sqrt{m^2 + V_0}$. The subscript I means that the solution is in the central region and all the above constants must be fixed by the boundary conditions. As we are interested in calculate resonances we must consider a plane wave coming from $-\infty$. This plane wave will collide with the membrane and will generate a reflected and a transmitted wave. Therefore for $z < -d/2$ we must have a linear combination of waves moving to the left and to the right. For $z > d/2$ we must have only one wave moving to the right. We define the solution in both regions by $\psi_L(z)$ and $\psi_R(z)$ respectively. In order to analyze resonances we fix the coefficient of the incoming wave equal to one. Therefore we choose

$$\begin{aligned} \psi_L(z) &= \sqrt{\frac{u}{m}} \left(H_\nu^{(2)}(u) + B_1 H_\nu^{(1)}(u) \right), \quad z < 0, \\ \psi_R(z) &= \sqrt{\frac{u}{m}} A_2 H_\nu^{(1)}(u), \quad z > 0. \end{aligned}$$

Defining $E(z) = \sqrt{\frac{u}{m}} H_\nu^{(2)}(u)$, $F(z) = \sqrt{\frac{u}{m}} H_\nu^{(1)}(u)$ for $z < 0$ and taking the continuity of the wave function and its derivative at $z = \pm d/2$, we obtain

$$\begin{pmatrix} E(-d/2) & F(-d/2) \\ E'(-d/2) & F'(-d/2) \end{pmatrix} \begin{pmatrix} 1 \\ B_1 \end{pmatrix} = \begin{pmatrix} e^{-iK\frac{d}{2}} & e^{iK\frac{d}{2}} \\ iKe^{-iK\frac{d}{2}} & -iKe^{iK\frac{d}{2}} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \quad (31)$$

and

$$\begin{pmatrix} F(-d/2) & 0 \\ -F'(-d/2) & 0 \end{pmatrix} \begin{pmatrix} A_2 \\ 0 \end{pmatrix} = \begin{pmatrix} e^{iK\frac{d}{2}} & e^{-iK\frac{d}{2}} \\ iKe^{iK\frac{d}{2}} & -iKe^{-iK\frac{d}{2}} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \quad (32)$$

After some algebra we finally obtain

$$A_2 = \frac{-4iK}{\pi(2KFF' \cos Kd + (F'^2 - F^2K^2) \sin Kd)}, \quad (33)$$

with the transmission coefficient given by

$$T = |A_2|^2 = \frac{16K^2}{\pi^2 |2KFF' \cos Kd + (F'^2 - F^2K^2) \sin Kd|^2}. \quad (34)$$

That is the final expression we use as starting point to make comparison between analytical and numerical methods. It is important to mention the number of iterations made using the method applied here. For calculations involving the dilaton contribution we have used 10^5 matrices, 2×10^5 matrices for the gravity case with $x = 1.5$, and 10^5 matrices with $x = 1.0$ for the scalar and gauge field cases. We have made another calculation of 2×10^5 matrices for the parameter $x = 0.15$ again for the vector gauge field resonances (x is the parameter describing the thickness of the membrane). For the tensor fields we computed 10^5 matrices for $x = 1.5$ and $x = 1.0$. For the case without the dilaton contribution, we have computed 3^5 matrices for $x = 1.5$ and 10^5 for $x = 1.0$ for the scalar field resonances. In the case of vector fields we made 10^5 iterations with $x = 1.0$ plus another computation of 10^5 matrices for $x = 0.15$. It is important to mention that we have found, for the case without dilaton, the same results for resonances in the gravity and scalar field cases. The results of these calculations will be discussed below.

4 The Gravity Case

As mentioned before the gravity case without dilaton has been considered previously in [36]. We will reconsider it here for completeness. The potential for this case is given by

$$\bar{U}_1(z) = \frac{3}{4} e^{2A_1(y)} (2A''(y) + 5(A_1'(y))^2). \quad (35)$$

The transformation to the z coordinate is obtained using $\frac{dz}{dy} = e^{-\bar{A}_1(z)}$ to get

$$\bar{U}_1(z) = \frac{3}{2} \bar{A}_1''(z) + \frac{9}{4} (\bar{A}_1'(z))^2, \quad (36)$$

what is equivalent to choose $b = 1$ and $c = 3/2$ in the previous section. From this we can obtain the solution taking $\nu = 2$.

The case with the dilaton coupling has the potential

$$\bar{U}_2(z) = \frac{3}{2}e^{3A_2(y)/2} \left(A_2''(y) + \frac{9}{4}(A_2'(y))^2 \right). \quad (37)$$

Where using the transformation $\frac{dz}{dy} = e^{-\frac{3}{4}\bar{A}_2(z)}$ we obtain

$$\bar{U}_2(z) = \frac{3}{2}\bar{A}_2''(z) + \frac{9}{4}(\bar{A}_2'(z))^2, \quad (38)$$

what is equivalent to choose $b = 3/4$ and $c = 3/2$. The solution is given by taking $\nu = 5/2$. Using now the expression for the transmission coefficient we obtain the results with dilaton in Fig. 6. In the figure we can see the appearance of one peak of resonance for the parameters $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. These peaks of resonance are very dependent on the parameter x , when we take $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$ the transmission coefficient for gravity with dilaton presents seven peaks of resonances shown in Fig. 7.

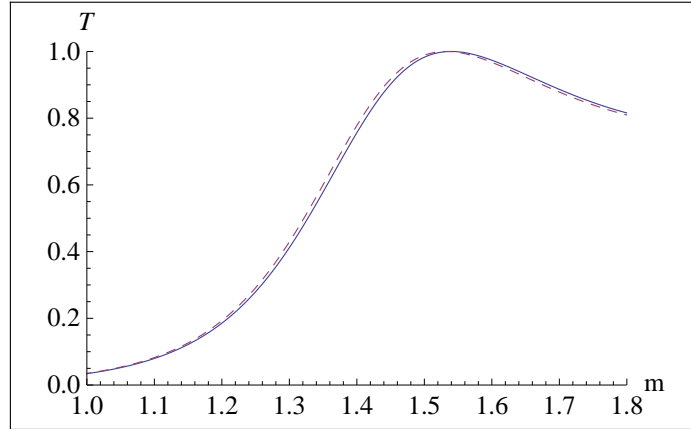


Figure 6: The transmission coefficient for gravity with dilaton for $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. The solid line is the analytical calculation and the dashed line is the numerical one.

5 The Scalar Field Case

Now we analyze the scalar field resonances by the analytical model proposed. As in the previous section we must obtain the potential in the z coordinate for the cases with and without the dilaton. These are given by

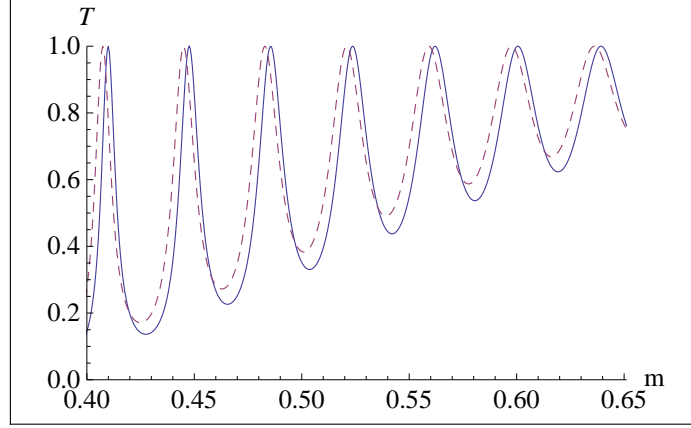


Figure 7: The transmission coefficient for gravity with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$. The solid line is the analytical calculation and the dashed line is the numerical one.

$$\bar{U}_1(z) = e^{3A_1(y)/2} \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)A_1(y)'' + \left(\frac{\alpha}{4} - \frac{9}{64}\right)(A_1'(y))^2 \right), \quad (39)$$

$$\bar{U}_2(z) = e^{2A_2(y)} \left(\frac{3}{2}A_2''(y) + \frac{15}{4}(A_2'(y))^2 \right), \quad (40)$$

where $\alpha = -15/4 - \lambda\sqrt{3M^3}$. Performing now the transformations $\frac{dz}{dy} = e^{-3\bar{A}_1(z)/4}$ and $\frac{dz}{dy} = e^{-\bar{A}_2(z)}$ respectively, we obtain

$$\begin{aligned} \bar{U}_1(z) &= \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)\bar{A}_1''(z) + \left(\frac{\alpha}{2} + \frac{3}{8}\right)^2(\bar{A}_1'(z))^2 \right) \\ &= \left(\left(\frac{3}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right)\bar{A}_1''(z) + \left(\frac{3}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right)^2(\bar{A}_1'(z))^2 \right), \end{aligned} \quad (41)$$

and

$$\bar{U}_2(z) = \left(\frac{3}{2}\bar{A}_2''(z) + \frac{9}{4}(\bar{A}_2'(z))^2 \right). \quad (42)$$

Therefore the solutions to both cases can be found by using

$$\begin{aligned} b_1 &= \frac{3}{4}, & c_1 &= \left(\frac{3}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right), \\ b_2 &= 1, & c_2 &= \frac{3}{2}, \end{aligned}$$

and we get

$$\begin{aligned}\nu_1^2 &= \left(\frac{5}{2} + \frac{2\lambda\sqrt{3M^3}}{3}\right)^2 \\ \nu_2 &= 2.\end{aligned}$$

We now compare these results with those obtained from the numerical method of transmission coefficients. We show in Fig. 8 the transmission coefficient for analytical (lined) and numerical (dashed) without dilaton with $x = 1.0$, showing one peak of resonance. In Fig. 9 with dilaton field we still have one peak of resonance for the parameters $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. In Fig. 10 we have the Transmission coefficient for scalar without dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$ and, in Fig. 11 showing eight peaks of resonances (the first one near $m = 0$, can be made more visible if we just give a zoom in that region), the Transmission coefficient for scalar with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$ showing three peaks.

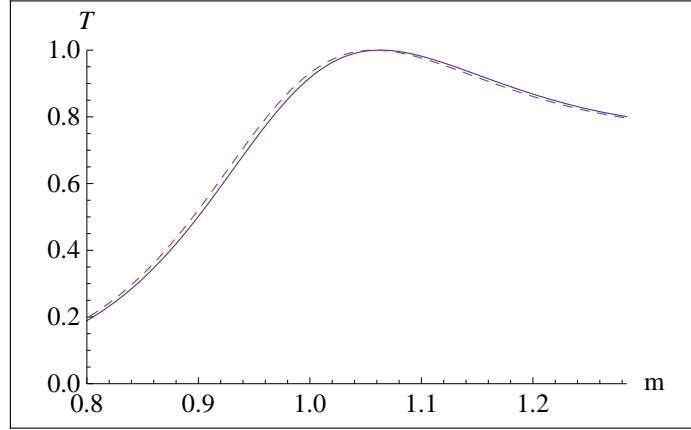


Figure 8: Transmission coefficient for scalar without dilaton for $x = 1.0$. Lined is analytic and dashed is numeric.

6 The Gauge Field Case

Now we turn our attention to analyze the gauge field resonances through the analytical context. Again we need to obtain the potential in the z coordinate for the cases with and without the dilaton. These potentials are given by

$$\bar{U}_1(z) = e^{3A_1(y)/2} \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)A_1''(y) + \left(\frac{\alpha}{4} - \frac{9}{64}\right)(A_1'(y))^2 \right), \quad (43)$$

$$\bar{U}_2(y) = e^{2A(y)} \left(\frac{1}{2}A''(y) + \frac{3}{4}(A'(y))^2 \right), \quad (44)$$

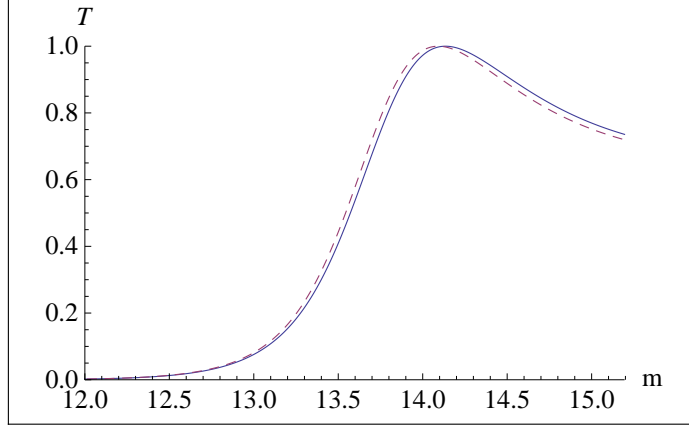


Figure 9: Transmission coefficient for scalar with dilaton for $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. Lined is analytic and dashed is numeric.

where $\alpha = -7/4 - \lambda\sqrt{3M^3}$. Performing now the transformations $\frac{dz}{dy} = e^{-3\bar{A}_1(z)/4}$ and $\frac{dz}{dy} = e^{-\bar{A}_2(z)}$ respectively, we obtain

$$\begin{aligned}\bar{U}_1(z) &= \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)\bar{A}_1''(z) + \left(\frac{\alpha}{2} + \frac{3}{8}\right)^2(\bar{A}_1'(z))^2 \right) \\ &= \left(\left(\frac{1}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right)\bar{A}_1''(z) + \left(\frac{1}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right)^2(\bar{A}_1'(z))^2 \right),\end{aligned}\quad (45)$$

and

$$\bar{U}_2(z) = \left(\frac{1}{2}\bar{A}_2''(z) + \frac{1}{4}(\bar{A}_2'(z))^2 \right). \quad (46)$$

Therefore the solutions to both cases can be found by using

$$\begin{aligned}b_1 &= \frac{3}{4}, & c_1 &= \left(\frac{1}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right), \\ b_2 &= 1, & c_2 &= \frac{1}{2},\end{aligned}$$

what gives for the solutions with and without the dilaton

$$\begin{aligned}\nu_1^2 &= \left(\frac{7}{6} + \frac{2\lambda\sqrt{3M^3}}{3}\right)^2, \\ \nu_2 &= 1.\end{aligned}$$

We show in Fig. 13 and in Fig. 12 the transmission coefficient for analytical (lined) and numerical (dashed) with and without dilaton with $x = 1.0$ showing

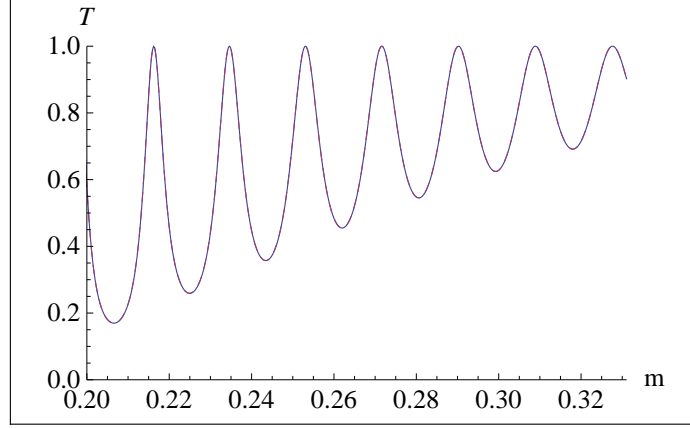


Figure 10: Transmission coefficient for scalar without dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$. Lined is analytic and dashed is numeric.

one peak of resonance, and in Fig. 14 we show the three peaks of the transmission coefficient for vector field with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$.

7 The Kalb-Ramond Case

The last bosonic field to be analyzed in five dimensions is the Kalb-Ramond field. Now we analyze this field resonances in the same way as in the last sections. The potential in the z coordinate for the cases with and without the dilaton are given by

$$\bar{U}_1(y) = e^{3A_1(y)/2} \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)A_1''(y) + \left(\frac{\alpha}{4} - \frac{9}{64}\right)(A_1'(y))^2 \right), \quad (47)$$

$$\bar{U}_2(y) = e^{2A_2(y)} \left(\frac{1}{2}A_2''(y) + \frac{3}{4}(A_2'(y))^2 \right), \quad (48)$$

where $\alpha = 1/4 - \lambda\sqrt{3M^3}$. Performing now the transformations $\frac{dz}{dy} = e^{-3\bar{A}_1(z)/4}$ and $\frac{dz}{dy} = e^{-\bar{A}_2(z)}$ respectively, we obtain

$$\begin{aligned} \bar{U}_1(z) &= \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)\bar{A}_1''(z) + \left(\frac{\alpha}{2} + \frac{3}{8}\right)^2(\bar{A}_1'(z))^2 \right) \\ &= \left(\left(-\frac{1}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right)\bar{A}_1''(z) + \left(-\frac{1}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right)^2(\bar{A}_1'(z))^2 \right), \end{aligned} \quad (49)$$

and

$$\bar{U}_2(z) = \left(-\frac{1}{2}\bar{A}_2''(z) + \frac{1}{4}(\bar{A}_2'(z))^2 \right). \quad (50)$$

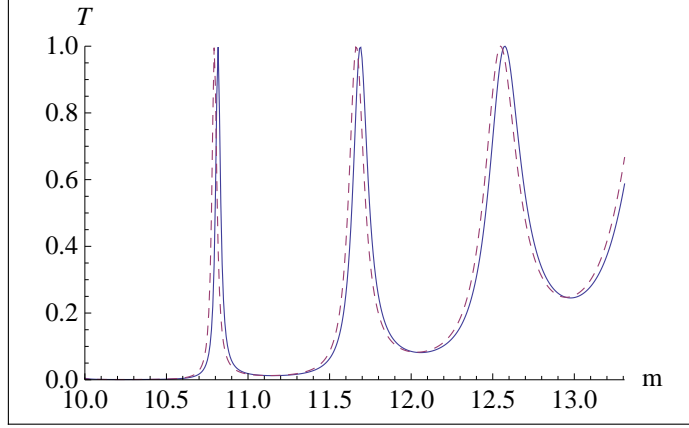


Figure 11: Transmission coefficient for scalar with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$. Lined is analytic and dashed is numeric.

Therefore the solutions to both cases can be found by using

$$\begin{aligned} b_1 &= \frac{3}{4}, & c_1 &= \left(-\frac{1}{2} + \frac{\lambda\sqrt{3M^3}}{2}\right), \\ b_2 &= 1, & c_2 &= -\frac{1}{2}, \end{aligned}$$

which gives for the solutions with and without the dilaton

$$\begin{aligned} \nu_1^2 &= \left(-\frac{1}{6} + \frac{2\lambda\sqrt{3M^3}}{3}\right)^2 \\ \nu_2 &= 0 \end{aligned}$$

The case without the dilaton has $a = -1/4$ and therefore will not be considered because it gives a negative potential.

We show in Fig. 15 the transmission coefficient for analytical (lined) and numerical (dashed) with dilaton with $x = 1.0$ and in Fig. 16 with $x = 1.5$. In both cases we have $\lambda\sqrt{3M^3} = 20$. In Fig. 15 we see just one peak of resonance around $m = 11.5$ and, in Fig. 16 we have got five peaks.

8 The q -form Case

The results of the last three cases can be summarized and generalized to a q -form in a p -brane, where $p = D - 2$. In a recent article the present authors have studied the resonances of these fields numerically in a different background [54].

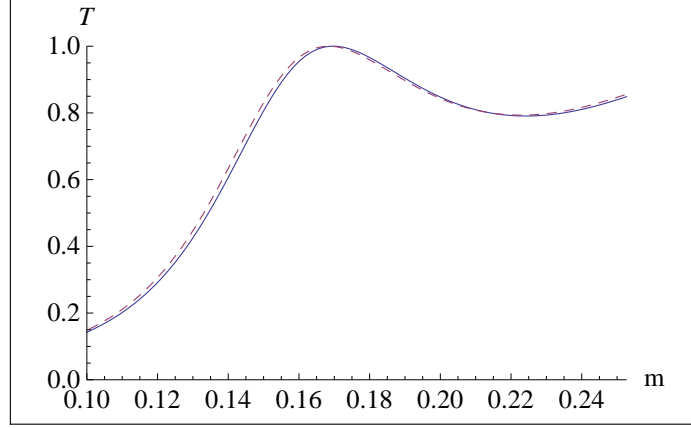


Figure 12: Transmission coefficient for vector field without dilaton for $x = 1.0$. Lined is analytic and dashed is numeric.

There the potential in the z coordinate for the case with the dilaton is given by

$$\bar{U}_1(z) = e^{3A_1(y)/2} \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)A_1''(y) + \left(\frac{\alpha}{4} - \frac{9}{64}\right)(A_1'(y))^2 \right). \quad (51)$$

In this equation $\alpha = (8q - 4p - 3)/4 - \lambda\sqrt{3M^3}$. Performing the transformations $\frac{dz}{dy} = e^{-3\bar{A}_1(z)/4}$ we obtain

$$\bar{U}_1(z) = \left(-\left(\frac{\alpha}{2} + \frac{3}{8}\right)\bar{A}_1''(z) + \left(\frac{\alpha}{2} + \frac{3}{8}\right)^2(\bar{A}_1'(z))^2 \right). \quad (52)$$

For the case without the dilaton, we have obtained the potential

$$\bar{U}_2(z) = e^{2A_2(y)} \left(\left(q - \frac{p}{2}\right)\left(q - \frac{p}{2}\right)(A_2'(y))^2 - \left(q - \frac{p}{2}\right)A_2''(y) \right), \quad (53)$$

and using the transformation $\frac{dz}{dy} = e^{-\bar{A}_2(z)}$ we obtain

$$\bar{U}_2(z) = \left(\left(\frac{p}{2} - q\right)\bar{A}_2''(z) + \left(\frac{p}{2} - q\right)^2(\bar{A}_2'(z))^2 \right). \quad (54)$$

Therefore the solutions for both cases can be found using

$$\begin{aligned} b_1 &= \frac{3}{4}, & c_1 &= \left(\frac{p}{2} - q + \frac{\lambda\sqrt{3M^3}}{2}\right), \\ b_2 &= 1, & c_2 &= \left(\frac{p}{2} - q\right), \end{aligned}$$

which gives the solutions for the cases with and without the dilaton

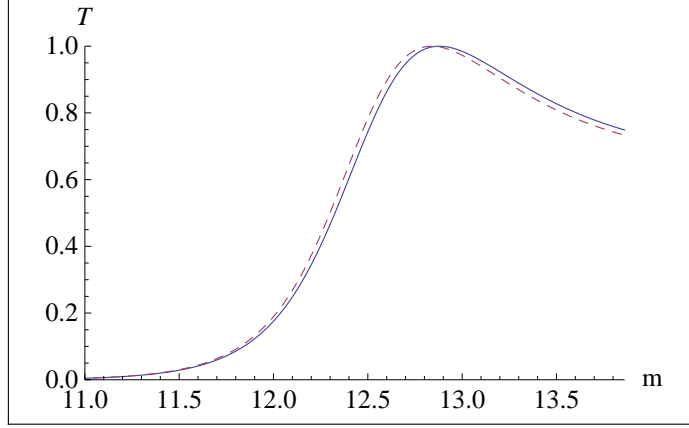


Figure 13: Transmission coefficient for vector field with dilaton for $x = 1.0$ and $\lambda\sqrt{3}M^3 = 20$. Lined is analytic and dashed is numeric.

$$\begin{aligned}\nu_1^2 &= \left(\frac{2\alpha}{3}\right)^2 \\ \nu_2^2 &= \left(\frac{1+p}{2} - q\right)^2.\end{aligned}$$

The above formulas can be verified for the cases $p = 3$ with $q = 0, 1, 2$ and agree with the last sections. For the case with the dilaton we show results for the parameter α only. The condition for positivity of the potential is $\alpha < -3/4$ or $\alpha > 3/4$. For the case without the dilaton we only have the parameter $(p/2 - q + 1/2)$. For this case the condition for the positivity of the potential is given by $q < p/2$ or $q > p/2 + 1$ and therefore we understand why, for the two form the potential is negative in $p = 3$. It is important to note that for $p > 3$ higher forms can be analyzed. We show in Fig. 17 the potential profile for a q -form field without dilaton with $p/2 - q = 5$.

For the q -form case the transmission coefficient is plotted in Fig. 18 for $x = 1.0$ and $\alpha = -1$ and in Fig. 19 for the same x and $\alpha = -2$ considering the dilaton. For the case without the dilaton the transmission coefficient is shown in Fig. 20 for $p/2 - q = 5$ and in Fig. 21 for $p/2 - q = 7$, $x = 1.0$ in both cases. One can see one peak of resonances in all figures except for Fig. 18 that has two peaks. For completeness, we show the transmission coefficient in Fig. 22 and Fig. 23 without dilaton for $p/2 - q = 5$ and $p/2 - q = 7$. They show six and eight peaks of resonances respectively.

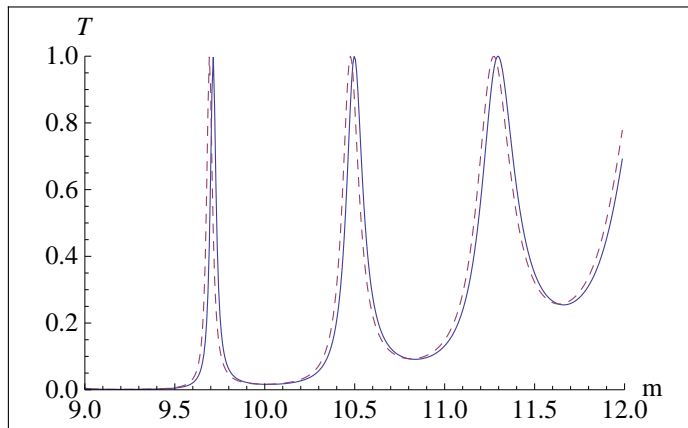


Figure 14: Transmission coefficient for vector field with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$. Lined is analytic and dashed is numeric.

9 Conclusions and Perspectives

In this work we have analyzed the transfer matrix method in the light of models with analytical solution. The background considered consist of a symmetric Z_2 thick domain wall interpolating between two BPS vacua. It has been shown previously that this background allows for an analytical solution of the gravity field modes [36]. By following the same lines of reasoning, as a first result we have found exact solutions to all the bosonic fields, namely, the scalar, the vector and the KR field. More than this, our solution to the Gravity case is slightly different of ref. [36]. The reason is that in that work they considered the wave function as real. Here we need a complex wave function since we consider plane waves arriving from the infinity and colliding with the membrane.

The first case used to test the numerical method was that of Ref. [55]. For this well known case the difference in the transmission coefficient values is of the order of 10^{-17} . For the other fields namely: the scalar field, the vector field, the Kb and q -form fields the agreement was impressively good. The results showed that the number of resonance peaks are very sensitive to the parameter x and the thickness of the membrane.

For the gravitational field we show Fig. 6. In that picture we can see the appearance of one peak of resonance for the parameters $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. However, in Fig. 7 we have the Transmission coefficient for gravity with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$ presenting seven peaks of resonances. This shows the rule of the x parameter in the setup chosen. We show for the scalar field, in Fig. 8, the transmission coefficient for analytical (lined) and numerical (dashed) without dilaton with $x = 1.0$, showing one peak of resonance. In Fig. 9 with dilaton field we still have one peak of resonance for the parameters $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. In Fig. 10 we have the Transmission coefficient for scalar

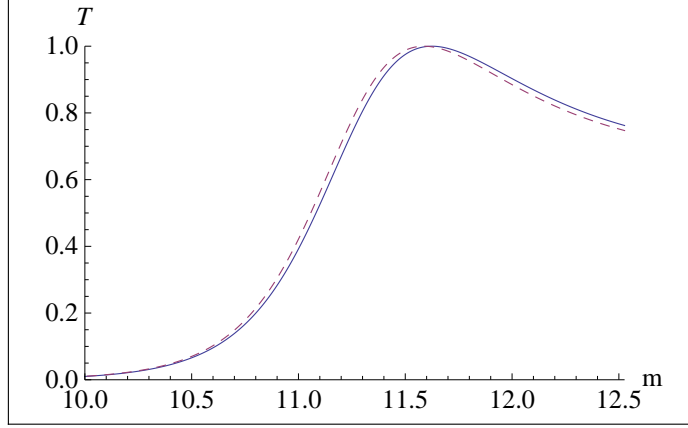


Figure 15: Transmission coefficient for tensor field with dilaton for $x = 1.0$ and $\lambda\sqrt{3M^3} = 20$. Lined is analytic and dashed is numeric.

without dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$ and, in Fig. 11 showing eight peaks of resonances (the first one near $m = 0$, can be made more visible if we just give a zoom in that region), the Transmission coefficient for scalar with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$ showing three peaks. For the case of the vector field, we show in Fig. 13 and in Fig. 12 the transmission coefficient for analytical (lined) and numerical (dashed) with and without dilaton with $x = 1.0$ showing one peak of resonance, and in Fig. 14 we show the three peaks of the transmission coefficient for vector field with dilaton for $x = 1.5$ and $\lambda\sqrt{3M^3} = 20$

In the case of the Kalb-Ramond field we show in Fig. 15 the transmission coefficient for analytical (lined) and numerical (dashed) with dilaton with $x = 1.0$ and in Fig. 16 with $x = 1.5$. In both cases we have $\lambda\sqrt{3M^3} = 20$. In Fig. 15 we see just one peak of resonance around $m = 11.5$ and, in Fig. 16 we have got five peaks. For the q -form case we give the graphics for the transmission coefficient in Fig. 18 and in Fig. 19 with $x = 1.0$ to $\alpha = -1, -2$ with dilaton and in Fig. 20 and Fig. 21 with $p/2 - q = 5, 7$ without dilaton. In the first one we can see two peaks of resonances, while in the remaining plots we have found just one peak. For completeness, we also give the graphics for the transmission coefficient in Fig. 22 and Fig. 23 with $p/2 - q = 5, 7$ without dilaton. They show, respectively, six and eight peaks of resonances.

The transfer matrix method has been extensively used by the present authors to analyze resonances. But it was important to compare the numerical studies with the analytical ones to show that the numerical method is reliable and have a perfect agreement with solvable cases. In here we give this comparison for a lot of cases which encloses the presentation of the method. There are still more cases to be studied like, for example, splitting membranes, a case which would just

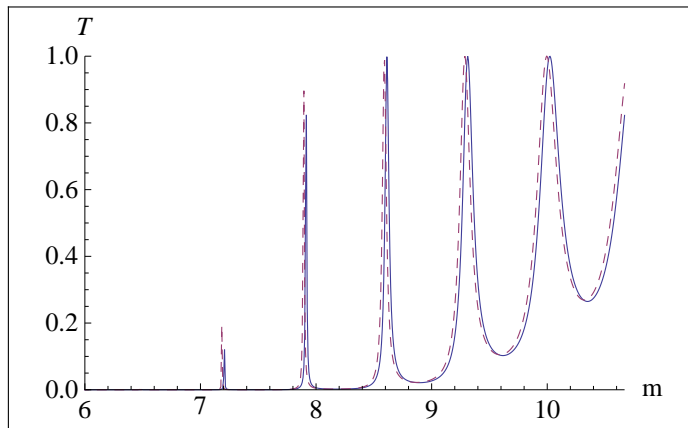


Figure 16: Transmission coefficient for tensor field with dilaton for $x = 1.5$ and $\lambda\sqrt{3}M^3 = 20$. Lined is analytic and dashed is numeric.

change the Schrödinger potential. The study of fermionic resonances within this formalism is also a perspective of the present work. Another important aspect is related to corrections obtained, for example, to the Newton's law. Although this is not the main goal of the present work, it is important to mention that these corrections can be obtained through the transmission coefficient. The issue of metastable states has been better studied in Refs. [20, 56]. Generally an expression is given in the literature which involves $\psi_m(0)$, however, as stressed in [56], this is obtained after the spectral density expansion in a complete set of modes. However, for our purpose it is better to keep the correction in terms of the spectral density, that is given by

$$V(r) \sim \int \frac{e^{-my}}{r} \rho(m) dm. \quad (55)$$

For a sharply peaked resonance we have $\rho = \delta(m - m_0)$ and we recover the well known formula for discrete masses [56]. When there is a peak in T , by current conservation there is also a peak in the spectral density and the above integration can be approximated by a sum. From the knowledge of the spectral density other important quantities as the width of the resonance can also be obtained.

Acknowledgment

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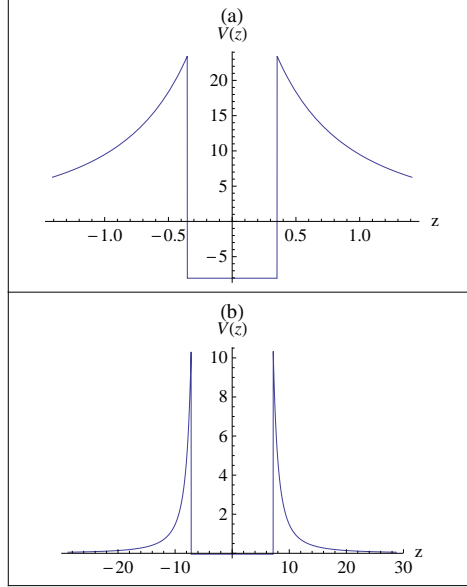


Figure 17: The potential profile for q -form field without dilaton with $p/2 - q = 5$. (a) $x = 1.0$ and (b) $x = 1.5$.

This paper is dedicated to the memory of my wife Isabel Mara (R. R. Landim).

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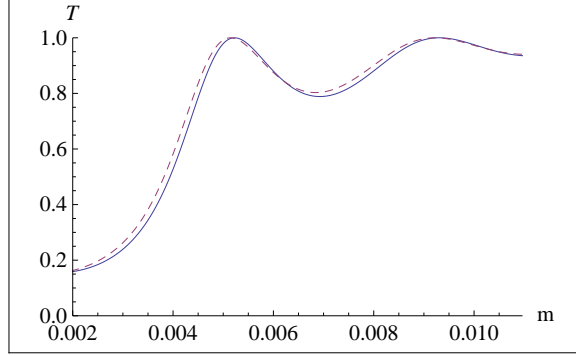


Figure 18: Transmission coefficient for q -form field with dilaton for $x = 1.0$ and $\alpha = -1$. The solid line is the analytical solution, the dashed line is the numerical solution.

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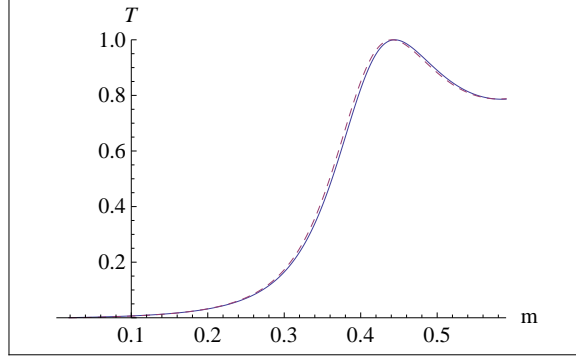


Figure 19: Transmission coefficient for q -form field with dilaton for $x = 1.0$ and $\alpha = -2$. The solid line is the analytical solution, the dashed line is the numerical solution .

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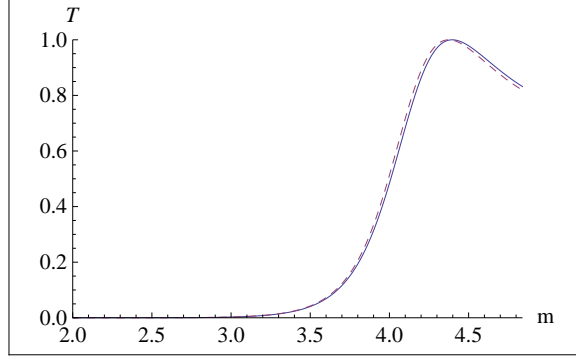


Figure 20: Transmission coefficient for q -form field without dilaton for $x = 1.0$ and $p/2 - q = 5$. The solid line is the analytical solution, the dashed line is the numerical solution .

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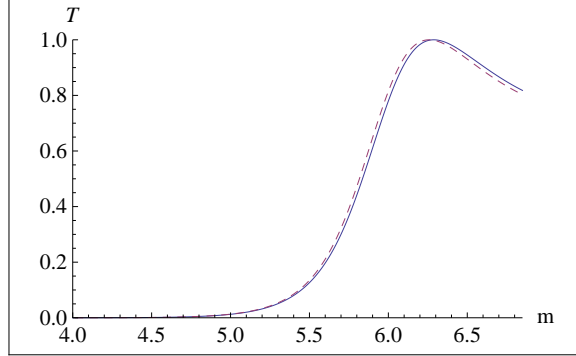


Figure 21: Transmission coefficient for q -form field without dilaton for $x = 1.0$ and $p/2 - q = 7$. The solid line is the analytical solution, the dashed line is the numerical solution .

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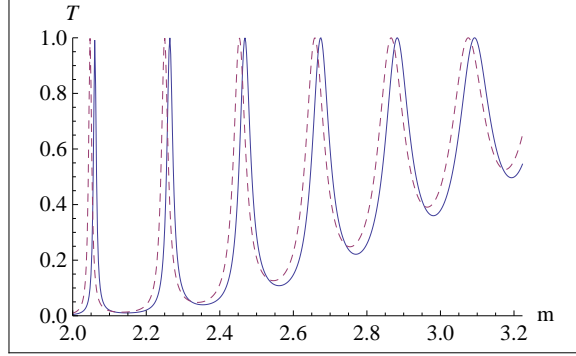


Figure 22: Transmission coefficient for q -form field without dilaton for $x = 1.5$ and $p/2 - q = 5$. The solid line is the analytical solution, the dashed line is the numerical solution .

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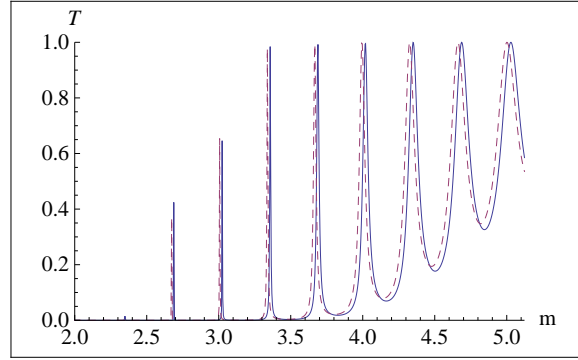


Figure 23: Transmission coefficient for q -form field without dilaton for $x = 1.5$ and $p/2 - q = 7$. The solid line is the analytical solution, the dashed line is the numerical solution .